EXERCISE – V JEE PROBLEMS

1. The radii r_1, r_2, r_3 of escribed circles of a triangle ABC are in harmonic progression. If its area is 24 sq. cm and its perimeter is 24 cm, find the lengths of its sides. **[REE 99, 6]**

2. (a) In a triangle ABC, Let $\angle C = \frac{\pi}{2}$. If 'r' is the inradius and 'R' is the circumradius of the triangle, then 2(r + R) is equal to **[JEE 2000 (Scr.), 1 + 1]**

(A) a + b (B) b + c (C) c + a (D) a + b + c

(b) In a triangle ABC, 2 ac sin$\frac{1}{2}$ (A – B + C) =

(A) $a^2 + b^2 - c^2$ (B) $c^2 + a^2 - b^2$

(C) $b^2 - c^2 - a^2$ (D) $c^2 - a^2 - b^2$

3. Let ABC be a triangle with incentre 'I' and inradius 'r'. Let D, E, F be the feet of the perpendiculars from 'I' to the sides BC, CA & AB respectively. If r_1, r_2 & r_3 are the radii of circles inscribed in the quadrilaterals AFIE, BDIF & CEID respectively, prove that

$$\frac{r_1}{r-r_1} + \frac{r_2}{r-r_2} + \frac{r_3}{r-r_3} = \frac{r_1 r_2 r_3}{(r-r_1)(r-r_2)(r-r_3)}$$ **[JEE 2000, 7]**

4. If Δ is the area of a triangle with side lengths a, b, c then show that : $\Delta \le \frac{1}{4}\sqrt{(a+b+c)abc}$. Also show that equality occurs in the above inequality if and only if a = b = c **[JEE 2001]**

5. Which of the following pieces of data does NOT uniquely determine an acute-angled triangle ABC (R being the radius of the circumcircle) ? **[JEE 2002 (Scr.), 3]**

(A) a, sin A, sin B (B) a,b,c

(C) a, sin B, R (D) a, sin A, R

6. If I_n is the area of n sided regular polygon inscribed in a circle of unit radius and O_n be the area of the polygon circumscribing the given circle, prove that

$$I_n = \frac{O_n}{2}\left(1+\sqrt{1-\left(\frac{2I_n}{n}\right)^2}\right)$$ **[JEE 2003, (Mains), 4]**

7. The ratio of the sides of a triangle ABC is $1 : \sqrt{3} : 2$. The ratio A : B : C is **[JEE 2004, (Scr.)]**

(A) 3 : 5 : 2 (B) $1 : \sqrt{3} : 2$

(C) 3 : 2 : 1 (D) 1 : 2 : 3

8. (a) In ΔABC, a, b, c are the lengths of its sides and A, B, C are the angles of triangle ABC. The correct relation is **[JEE 2005, (Scr.)]**

(A) $(b-c)\sin\left(\frac{B-C}{2}\right) = a\cos\left(\frac{A}{2}\right)$

(B) $(b-c)\cos\left(\frac{A}{2}\right) = a\sin\left(\frac{B-C}{2}\right)$

(C) $(b+c)\sin\left(\frac{B+C}{2}\right) = a\cos\left(\frac{A}{2}\right)$

(D) $(b-c)\cos\left(\frac{A}{2}\right) = 2a\sin\left(\frac{B+C}{2}\right)$

(b) Circles with radii 3, 4 and 5 touch each other externally if P is the point of intersection of tangents to these circles at their points of contact. Find the distance of P from the points of contact. **[JEE 2005 (Mains), 2]**

9. (a) Given an isosceles triangle, whose one angle is 120º and radius of its incircle is $\sqrt{3}$. Then the area of triangle in sq. units is **[JEE 2006, 3]**

(A) $7+12\sqrt{3}$ (B) $12-7\sqrt{3}$ (C) $12+7\sqrt{3}$ (D) 4π

(b) Internal bisector of $\angle$A of a triangle ABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F. If a, b, c represent side of ΔABC then **[JEE 2006, 5]**

(A) AE is HM of b and c (B) AD = $\frac{2bc}{b+c}\cos\frac{A}{2}$

(C) EF = $\frac{4bc}{b+c}\sin\frac{A}{2}$ (D) the triangle AEF is isosceles

10. Let ABC and ABC' be two non-congruent triangles with sides AB = 4, AC = AC' = $2\sqrt{2}$ and angle B = 30º. The absolute value of the difference between the areas of these triangles is **[JEE 2009, 5]**

11. In a triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\sin^2 A/2$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C, respectively, then
(A) $b + c = 4a$ (B) $b + c = 2a$ **[JEE 2009]**
(C) locus of point A is an ellipse
(D) locus of point A is a pair of straight lines

12. If the angles A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is
(A) 1/2 (B) $\sqrt{3}/2$ (C) 1 (D) $\sqrt{3}$ **[JEE 2010]**

13. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are)
(A) $-\left(2+\sqrt{3}\right)$ (B) $1+\sqrt{3}$ **[JEE 2010]**
(C) $2+\sqrt{3}$ (D) $4\sqrt{3}$

14. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to **[JEE 2010]**

15. Let PQR be a triangle of area Δ with a = 2, b = $\frac{7}{2}$ and c = $\frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals
(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$ **[JEE 2012]**